Evaluating Strategic Play: Manipulations with Symmetric Information in the Boston Mechanism

by

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Abstract

The Boston mechanism, present in educational, medical, and military settings, is highly manipulable by sophisticated agents using knowledge about which choices are popular. However, if agents are unaware of the relative popularity of choices, instead holding symmetric information about others' reports and priority rankings, there can still be theoretical benefits to manipulating. We introduce the quasi-sophisticated strategy as a heuristic that agents can use to decide whether to report truthfully or reverse the ordering of two choices under symmetric information. We evaluate the benefits of following this strategy, and compare it to the expected utility of truth-telling and of having full information about others' preferences. We do this using data from the Educational Studies program, which assigns students to classes using a version of the Boston Mechanism. We find that the quasi-sophisticated strategy performs worse than truth-telling by an amount equivalent to a 3.4 percentage point decrease in the likelihood of enrollment into ones' first choice. This inferior performance is driven by both classes that are under-demanded and heterogeneity in the popularity of classes. These results indicate that in the Boston mechanism, if students lack information about the relative popularity of classes, sophisticated agents may be best off by simply being truthful.

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1 Background

The design of centralized market-clearing mechanisms, used to assign physicians to residency programs, army cadets to branches, students to schools, and more, can have far-reaching consequences for the mechanism's participants. In recent years, economists have extensively studied manipulable mechanisms, in which a participant can benefit by misreporting their true preferences, and how agents who manipulate affect the assignments of all participants. Many manipulations require information about other participants' reports; for example, in the Boston mechanism (Abdulkadiroğlu and Sönmez, 2003), students have an incentive to not report their true first-choice school if they have low priority at that school and if they expect many other students to desire that school and. However, students may not have perfect knowledge about schools' priority rankings or other students' preferences. Is there still a benefit to manipulating if a student has incomplete information?

In this paper, we describe a strategy that students can use to manipulate the Boston mechanism if they have symmetric information about other students' preferences and school priorities are randomly assigned. This strategy, which we term the *quasi-sophisticated strategy*, implies that students should reverse their first and second choice if the ratio of utility exceeds the reciprocal of the ratio of their capacities. This strategy assumes that schools are all overdemanded, and so cannot accommodate all students who rank it first, and is a reasonable approximation of the strategy taken by a fully-rational agent with symmetric information in highly oversubscribed settings.

To evaluate the performance of this strategy against both truth-telling and having full information, we use a novel data set from the Educational Studies Program (ESP). ESP is a student organization at the Massachusetts Institute of Technology that manages multiple educational programs for students in grades 7-12. At these programs, students are assigned to multiple classes using a version of the Boston mechanism modified for a many-to-many matching market. Using 3.36 million simulations across 28 ESP programs between 2014 and 2020, we simulate the outcomes of 16,800 hypothetical students who have preferences over two class sections, and are choosing whether to report their preferences truthfully, follow the quasi-sophisticated strategy with symmetric information, or be fully rational if they have full information about others' reports. We find that truth-telling outperforms the quasi-sophisticated strategy by 0.036 units, an amount equivalent to increasing a students' likelihood of enrollment in their first choice by 3.6 percentage points. This poor performance is driven by the fact that 45.6% of class sections have enough capacity to accommodate all students who rank it first, which does not align with the "overdemanded" assumption made by a student following the quasi-sophisticated strategy. If we limit analysis to pairs of overdemanded class sections, a subsample which is more suited for the quasi-sophisticated strategy, the strategy still does no better than truth-telling in expectation, a null result which is driven by heterogeneity in students' reports. These results suggest that even in highly manipulable mechanisms such as the Boston mechanism, students may face individual incentives to be sincere in their reports if they are uncertain about the relative popularity of different classes.

This paper has three distinct contributions to the literature. First, our theoretical model extends the work on symmetric information as introduced by Roth and Rothblum (1999), a paper that concludes workers who hold symmetric information and are assigned to firms using deferred acceptance (Gale and Shapley, 1962) can only benefit by truncating their true preferences in their reports, and cannot benefit by "reversing" the order of two firms in their report. Ehlers (2008) extends this work to many-to-one markets for any mechanism that satisfies four properties, including all priority mechanisms. This paper is the first to examine strategic misreporting by an agent with symmetric information in a many-to-one market when firms (or schools or classes) have varying capacities.[1]

[1] Theorem 3.1 in Ehlers (2008), shows that a student in a many-to-one matching market that uses the

Second, this paper furthers research on the role of information in school choice mechanisms. Hastings and Weinstein (2008) examine how providing test-score results to disadvantaged families alters the choices that they make in a school assignment mechanism in the Charlotte-Mecklenburg Public School District. Kapor et al. (2020) discuss strategic play when students are misinformed about the likelihood of admission into different schools using data from New Haven. In this paper, we show that even in a mechanism that is highly manipulable for agents with full information, a truth-telling student with symmetric information (and thus false beliefs about their admissions likelihood into classes), is *harmed* by choosing to act quasi-strategically.

Third, this paper contributes to the literature on manipulations in the Boston mechanism, as introduced by Abdulkadiroğlu and Sönmez (2003). Abdulkadiroğlu et al. (2006) provide evidence of strategic and sophisticated behavior in the Boston mechanism using data from Boston Public Schools, using the fact that a sophisticated student would never rank overdemanded schools as their second (or later) choice. Pathak and Sönmez (2008) provide theoretical results about equilibria in the Boston mechanism when some students are sophisticated and some are sincere, showing that sophisticated students benefit at the expense of sincere students. Dur et al. (2018) uses this theoretical result to estimate the benefits of sophisticated behavior in the Wake County Public School system. In this paper, we show that while a student with symmetric information can act strategically in theory, there is no empirical gain to such behavior.

The rest of the paper is organized as follows. Section 2 introduces the Boston mechanism and describes when a quasi-sophisticated agent with symmetric information will misreport their true preferences. Section 3 discusses the Educational Studies Program and the ESP mechanism. Section 4 details our methodology and results. Section 5 concludes.

Boston mechanism cannot benefit by exchanging the ordering of two firms when reporting her preferences. This theorem requires an "anonymity" condition to hold, but this is not satisfied if firms vary in capacity.

2 Theory

2.1 The Boston Mechanism

Consider the traditional school choice problem with the Boston mechanism, as introduced by Abdulkadiroğlu and Sönmez (2003). In this environment, students seek enrollment in a school for the upcoming school year. Each student has preferences over the set of schools and being unassigned. Schools have a maximum capacity of students that they can admit, and also have a strict priority ordering over the set of students.

Formally, this economy consists of:

- students $I = \{i_1, i_2, \ldots, i_n\}$,
- schools $S = \{s_1, s_2, \ldots, s_m\}$,
- school capacities $q = \{q_{s_1}, q_{s_2}, \ldots, q_{s_m}\}$,
- school priorities $\mathcal{R}_S = \{R_{s_1}, R_{s_2}, \ldots, R_{s_m}\}$ where each R_s is school s's priority ranking over the set of students I, a ranking that is set exogenously by a district or state, and
- student preferences $\mathcal{R}_I = \{R_{i_1}, R_{i_2}, \ldots, R_{i_n}\}$, where each R_i is the preference of student i over the set $S \cup \{i\}$; a student's preference for themself represents a preference for being unmatched.

For convenience, let $R_s(k)$ and $R_i(k)$ represent the k^{th}-priority student at school s and the school ranked k^{th} by student i, respectively.

A matching in this economy is an assignment of students to schools, and can be represented as a function $\mu : I \rightarrow S \cup I$ that maps students to their assignments and satisfies:

- For all $i \in I$, $\mu(i) \in \{i\} \cup S$
- For all $s \in S$, $|\mu(s)^{-1}| \leq q_s$

In the Boston mechanism, a matching μ is computed using $I, S, q, \mathcal{R}_S$, and students' reported preferences $\widehat{\mathcal{R}_I} = \left\{\widehat{R_{i_1}}, \widehat{R_{i_2}}, \ldots, \widehat{R_{i_n}}\right\}$, which is not necessarily the same as students' true preferences $\mathcal{R}_I$. Given the 5-tuple $\langle I, S, q, \mathcal{R}_S, \widehat{\mathcal{R}_I}\rangle$, the Boston mechanism computes a matching using the following algorithm:

> Step 1. For each school s, consider only the students who ranked it first under $\widehat{\mathcal{R}_I}$, in other words, $\{i \mid \widehat{R_i}(1) = s\}$. Among these students, assign students a seat at s one at a time according to the priority order $\mathcal{R}_S$ until either there are no more seats at s, or we have assigned all students who ranked s as their first choice to s.
>
> Step k: For each school s with seats remaining, consider all students who are still unassigned who ranked s as their k^{th} choice in $\widehat{\mathcal{R}_I}$, in other words, in other words, $\{i \mid \widehat{R_i}(k) = s\}$. Assign these students to s according to the priority order $\mathcal{R}_S$ until either there are no more seats at s, or we have assigned all students who ranked s as their k^{th} choice to s.

This algorithm proceeds until all students have been assigned.

2.2 Reversal Strategies in the Boston Mechanism

Much literature has focused on the manipulability of the Boston mechanism if agents are aware of others' preferences and schools' priority rankings. Because of the "immediate acceptance" nature of the Boston mechanism, a student whose true first choice is A, a highly-demanded school that they do not have priority at, is not only unlikely to be enrolled in A if they report it first, but is also be unlikely to be assigned to any *other* school because they would be considered after students that ranked other schools first. As a result, a student may choose to misreport their true preferences, instead ranking first a less popular school at which they have priority.

However, students may not have access to complete information about $\mathcal{R}_S$ and $\widehat{\mathcal{R}_I}$; for example, they may be unaware of the popularity of a new school, they may not be able to gather information perfectly, or schools may hide their priority ranking from the public. However, even when a student possesses very little information about others' preferences and schools' priority rankings, it may still be may be advantageous to manipulate. We demonstrate this with the following example.

Example 1. Suppose that there are 4 students, $I = \{i_1, i_2, i_3, i_4\}$ and 2 schools, $\{s_1, s_2\}$, with capacities 1 and 2, respectively. Student i_1 has utility function U that is linear in expectation, with $U(s_1) = 1$, $0 < U(s_2) < 1$, and $U(i_1) = 0$. She is unaware of other students' preferences, and believe that other students are equally likely to report $s_1 \succ s_2$ as $s_2 \succ s_1$, and do so independently. She is also unaware of each school's priority ranking, and believes that R_{s_1} and R_{s_2} are both random orderings of I. We represent the expected outcomes of i_1 when she reports her true preferences $s_1 \succ s_2$, conditional on other students' preferences, and expected utilities if she reports reversed preferences $s_2 \succ s_1$, in Table 1.

In Table 1, rows 1 and 4 each occur with probability $\frac{1}{8}$ each, while rows 2 and 3 occur with probability $\frac{3}{8}$; as such, the expected utility of truth-telling is $\frac{1}{32}(6U(s_2) + 15)$ while her expected utility for reporting her preferences in reverse order is $\frac{1}{32}(26U(s_2) + 1)$. As a result, whenever $U(s_2) > \frac{7}{10}$, it is advantageous for her to misrepresent her preferences.

The intuition behind this result is that "no information" is still information when viewed in the context of school capacities, and so can be used to manipulate. As student i_1 believes that each class will be equally popular, her more preferred class is "risky" compared to her second choice option with higher capacity. If her utility for receiving her less-preferred option is sufficiently high, there is a benefit (in expectation) to misreporting her preferences. □

In Example 1, the steps required to analyze all possibilities increases exponentially as the number of schools and the length of students' preference lists increase. As a result, a cognitively-limited agent will be unable to fully evaluate their utilities under these two strategies.[2] Instead, suppose that an agent follows a *quasi-sophisticated strategy* in which she assumes she believes that every school will fill its capacity with just first choices, and so she will never be enrolled in her second, third, etc., choices. This allows the student to simplify their analysis, because they can ignore the calculations needed to compute their likelihood of enrollment in choices beyond their first one. We demonstrate the quasi-sophisticated strategy in Example 2.

Example 2. Consider the same setup from Example 1. Student i_1 has symmetric information about the rankings of the other 3 students and the 2 schools. She chooses whether to report her true preferences or reversed preferences assuming that she is enrolled into schools beyond her first rank with probability 0. Under these beliefs, she calculates her expected utility using the quantities in Table 2.

Again, we weight each row by the probability of its occurrence to compute her overall expected utility. In this case, i_1 believes her utility for reporting truthfully is $\frac{15}{32}$, while her utility for reversing her preferences is $\frac{13}{16}U(s_2)$. She will then choose to misreport if and only if $U(s_2) > \frac{15}{26}$, a cutoff about 17.6% less than her cutoff when she was fully sophisticated. Here, the quasi-sophisticated strategy is a reasonable (albeit not fully accurate) heuristic, that i_1 can use to evaluate when she should misreport her preferences. □

[2] The behavior of cognitively-limited agents is discussed by Li (2017) and Troyan and Morrill (2020), who use this to define if mechanisms are "obviously" strategy-proof and manipulable, respectively.

To formalize this intuition of these two examples, we define "symmetric information" in the same manner as Roth and Rothblum (1999) and Ehlers (2008). Let $R_i^{s\leftrightarrow s'}$ be the preference ordering that is identical to R_i but with the positions of s and s' exchanged. Let $R_{-i}^{s\leftrightarrow s'}$ denote the preference profile obtained by exchanging of s and s' in the preferences of all students except i and also exchanging the priorities of school s and s'.

Suppose that student i believes that school priorities and other students' stated preferences, $\mathcal{R}_{-i} = \mathcal{R}_S \cup \widehat{\mathcal{R}_I} \setminus \{R_i\}$, are drawn from some distribution, and so is given by the random vector $\widetilde{\mathcal{R}}_{-i}$, termed by Roth and Rothblum (1999) as a random preference profile. Student i may have uncertainty about others' reports; one way to model this uncertainty is to say that i holds $\{s, s'\}$-symmetric information about $\widetilde{R}_{-i}$, such that for any report $\mathcal{R}_{-i}$ that others give, $\Pr\left(\widetilde{\mathcal{R}}_{-i} = \mathcal{R}_{-i}\right) = \Pr\left(\widetilde{\mathcal{R}}_{-i} = \mathcal{R}_{-i}^{s\leftrightarrow s'}\right)$. In essence, i believes it to be equally likely other that students prefer s' to s or s to s' and that i is unable to distinguish between the priorities of s and s'.[3]

Define the quasi-sophisticated strategy as the optimal strategy taken by an agent who believes that each school is overenrolled in Step 1 of the Boston mechanism — using the terminology of Abdulkadiroğlu et al. (2006), we say that this agent believes that all schools are *overdemanded*. Formally, the quasi-sophisticated strategy is the strategy taken by an agent who believes that $\Pr\left(\left|\left\{i' \mid \widehat{R_{i'}}(1) = s,\ i' \neq i\right\}\right| \geq q_s\right) = 1$ for all schools s. This assumption implies that the agent is either either enrolled into their first choice or unenrolled, and that the probability of enrollment into his second, third, etc., choices is 0.

There are two ways to interpret the quasi-sophisticated strategy. First, the quasi-sophisticated strategy is a way for an agent to evaluate potential strategies without computing all possibilities for enrollment into lower-ranked choices. While it may not always be fully

[3]Symmetric information does not imply that class section popularity is independent. As an example, an agent who believes that with 50% probability, all students rank $s_1 \succ s_2$, and with 50% probability, all students rank $s_2 \succ s_1$, and also believes that school priorities are a random ordering of I still has symmetric information about those two schools.

accurate (as in Example 2), it is a simple heuristic that an agent can use to choose between different reports. Second, if all classes are *actually* overdemanded, the quasi-sophisticated strategy is identical to the strategy taken by a fully-rational. See Appendix C for an example akin to Examples 1 and 2 where this condition is approximately true.

The following theorem describes the behavior of a quasi-sophisticated student who has symmetric information under the Boston mechanism and believes that school priorities are randomly chosen.

> **Theorem 1.** Consider a school choice market where in which seats are assigned using the Boston mechanism. Suppose that i has $\{s, s'\}$-symmetric information, that i believes that s and s' are overdemanded, and that each school's priority ranking R_s is a random ordering of I. If i has preferences $s \succ s'$, derives no utility from being unmatched, and follows the quasi-sophisticated strategy, she will choose to rank s first if and only if $U(s)q_s \geq U(s')q_{s'}$. Else, she benefits by reporting s' first.

Proof. Let X_s be a random variable indicating the number of students that school s is considering in the first round (all of whom ranked s first), excluding i. If i also ranks s first, she expects to be enrolled with probability $\Pr(\mu(i) = s) = E\left[\min\left(\frac{q_s}{X_s+1}, 1\right)\right]$ because R_s is a random ordering of all students, and so the probability of enrollment is either the number of available spots divided by the number of people who apply for s, or 1 if the class is undersubscribed. However, the "overdemanded" assumption implies that $\Pr(X_s + 1 \geq q_s) = 1$ and so the class will never be undersubscribed. As such, we have that $\Pr(\mu(i) = s) = q_s \cdot E\left[\frac{1}{X_s+1}\right]$, and so her expected utility from ranking s first is $q_s U(s) E\left[\frac{1}{X_s+1}\right]$. Because she follows the quasi-sophisticated strategy, we need not consider the utility gained from being enrolled in any lower-ranked schools. The same logic applies to s'.

Comparing the utilities of ranking s first and of ranking s' first, we conclude that student i is truthful about her first choice if and only if $q_s U(sE\left[\frac{1}{X_s+1}\right] \geq q_{s'} U(s')E\left[\frac{1}{X_{s'}+1}\right]$. As the distributions of X_s and $X_{s'}$ coincide because of $\{s, s'\}$-symmetric information, this condition is equivalent to $U(s)q_s \geq U(s')q_{s'}$.

□

Theorem 1 provides a cutoff under which a student will choose to misreport his preferences under the Boston mechanism in a many-to-one matching market. However, this theorem makes no guarantees about the magnitude of utility gained by following this strategy. In particular, if schools are not actually overdemanded, the quasi-sophisticated strategy incorrectly estimates the likelihood of enrollment into classes, which may be detrimental to students. Further, if a student has $\{s, s'\}$-symmetric information about others' preferences, but the realization of others' preferences is not $\{s, s'\}$-symmetric, a student may lose utility in expectation by following the quasi-sophisticated strategy.

3 Empirical Setting

3.1 The Educational Studies Program

To evaluate the viability of the quasi-sophisticated strategy in a real matching market, we use data from the MIT Educational Studies Program (ESP). ESP is a student-run organization that has managed educational events for middle and high school students since 1957. ESP's motto is "Teach Anything, Learn Anything"; at ESP's programs, college students and members of the greater MIT community teach classes to students on any topic they wish, with class titles such as "Conceptual Quantum Mechanics", "The Mathematics of Music", "Hacking Your Memory", "How to Figure Skate in Socks", and "Appreciation for any Juice which is not Ruby Red".[4] (ESP, 2021)

Each program consists of tens or hundreds of class sections scheduled across several days. Teachers have full autonomy over every element of their section, including scheduled time, capacity, subject material, and class style. Teachers are permitted to teach multiple sections of the same parent class, each with the same content.[5] Hundreds or thousands of students participate in these programs, learning about program registration through email lists, school outreach, and social networks. While a majority of students come from the greater Boston area, some travel from further away to participate. Teachers have no control over which students are enrolled in their class, beyond limiting their class to specific grades.

Our data spans from 2014 to 2020 and includes all ESP programs (Spring HSSP, Summer HSSP, Spark, and Splash) which use the ESP mechanism to assign students to classes, as described in Subsection 3.2. Our data consists of all classes in our programs of interest that had at least one student who indicate interest, and excludes lunch "classes", used by ESP to provide meals to students. Summary statistics about class sections at ESP programs are included in Table 3.

The smallest programs, as measured by student and class section count, are instances of Spring and Summer HSSP. HSSP is a multi-weekend program where students take the same classes each weekend. Spark, the second-largest program, is a one-weekend "learning extravaganza" in March, at which middle-school students can enroll in a large variety of classes. Splash, the largest program, follows the same format as Spark, but occurs in November and is designed for high-school students.

While there is a large variance in number of students and classes across programs, almost

[4] Two of these were taught by the author.

[5] We use the term class section to refer to one instance of a parent class. Our focus will be on preferences over class sections, as discussed further below. We use the terms "class section", "section", and "class" interchangeably. We will only use the term "parent class" to refer to parent classes.

every program is oversubscribed, as measured by the "supply" of student-class hours (the sum of each class section's capacity multiplied by its duration) divided the "demand" for student class-hours (the number of students multiplied by the number of available timeblocks). If this ratio is less than 1, it is impossible in theory for every student to have a full schedule of classes throughout the entire program. If schedule and grade constraints are taken into consideration, then a ratio larger than 1 is needed to ensure that every student has a full schedule. The most oversubscribed programs, Spring HSSP 2016 and Summer HSSP 2015–2019, have supply-demand ratios below 0.5, indicating that at best, every student could have a half-filled schedule.

Formally, an ESP program has students I, class sections S, capacities q, and student preferences $\mathcal{R}_I$ as defined as in Subsection 2.1 for the Boston mechanism. In addition, each program occurs over a series of ordered and non-overlapping timeblocks, given by $T = \left\{[t_1, t'_1), [t_2, t'_2), \cdots, [t_{|T|}, t'_{|T|})\right\}$, where $t_1 < t_2 < \cdots < t_{|T|}$ and $[t_j, t'_j) \cap [t_k, t'_k) = \emptyset \; \forall \; j, k$. Class sections are scheduled according to $D = \{D_{s_1}, D_{s_2} \cdots, D_{s_m}\}$, where each $D_i \subset T$ is the set of timeblocks in which c_i occurs. The resulting many-to-many matching problem is given by the 6-tuple $\langle I, S, q, T, D, \mathcal{R}_I \rangle$.

3.2 The ESP Mechanism

At each program, ESP computes an assignment of students to classes. This matching enrolls each student i in a subset of class sections S that has no schedule overlap and does not contain two class sections belonging to the same parent class, and also so that no class section is overenrolled. Formally, a matching μ is a correspondence $\mu : I \rightarrow S \cup I$ such that:

- For all $s \in S$, $|\mu(s)^{-1}| \leq q_s$
- For all $i \in I$, either $\mu(i) = i$ (unmatched), or $\mu(i) \subseteq S$ such that for all $s_j, s_k \in \mu(i)$, $j \neq k$, $D_{s_j} \cup D_{s_k} = \emptyset$ (no overlap in class times) and s_j and s_k do not belong to

the same parent class.[6]

At ESP programs, students do not directly report a ranking over all class sections. Instead, for each timeblock t students can rank one class section that starts at t.[7] In addition, each parent class can be designated with a "star", indicating interest in the class as a whole, with no limit on the number of stars assigned.[8] This star gets applied to each section of a parent class.

The ESP Matching Mechanism, in a way similar to the Boston mechanism, computes a matching in the environment described above, using the following steps:

Step 1. Assign each student a weight factor of 1.

Step 2. Randomize the list of class sections S such that sections which occur over more time blocks are earlier in the ordering than shorter classes (length as defined by $|D_s|$).[9]

Step 3. For each section s in S (in order), consider only students that listed it as Rank 1 and have all timeblocks of s available (i.e., not enrolled in a conflicting class). If the section has enough capacity to accommodate all students, then assign seats to all of those students. Else, randomly select as many students as possible to fill the class, choosing students with a probability proportional to their weight factor.[10] For each student enrolled in s, divide their weight factor by 1.2 for future section assignments.[11]

[6]The "parent class" condition is easily accounted for using the timeblock constraint. By inserting "indicator timeblocks", one for each parent class, after all real timeblocks, and augmenting each D_s with the timeblock associated with the parent class of s, students will never be enrolled in two instances of the same parent class. We mention parent classes in our setup because of the nature of the ESP mechanism, described below.

[7]In the programs we study, students were actually able to rank 3 classes; however, a coding error caused the mechanism to ignore second and third choices. This error was discovered in the course of this research, and has since been corrected.

[8]A student can both star a class and rank it as their first choice.

[9]This step is performed to ensure a smoother distribution of students between class sections of different length.

[10]To be precise, the mechanism, implemented in Python, uses `numpy.random.choice`.

[11]The nature of this "tiebreaker" helps to ensure more equitable class assignments across students, as students who have already been enrolled in classes will be less likely to be enrolled in others.

Step 4. Repeat steps 2 and 3, but considering students who starred each class section, and use the weight 1.1.

This mechanism is "Boston-like", in the sense that students are either accepted or rejected in the round that a class section considers them. In the case where there is only one timeblock T_1, student weights become irrelevant, as students can only be enrolled in exactly one class, and the ESP mechanism reduces exactly to the Boston mechanism with randomly chosen school priorities. Similarly, if a student has preferences only over classes with the same duration that occur in the same timeblocks, that student faces a problem identical to the Boston mechanism. In general, the ESP mechanism faces similar incentives as does the Boston mechanism; in particular, if a student believes a class to be sufficiently popular, it may be beneficial to rank a less-preferred class first, rather than "waste" one's ranked class on their true first choice that is in high demand.

Consider a student i who only has preferences for two class sections s and s' which have the same length and start in the same timeblock. Class sections s and s' are treated "equally" by the lottery. If i is fully rational, her choice about whether to rank s or s' first is dependent on how many people she expects to be eligible for enrollment into s and s' in Step 3 of the above algorithm (ranked s or s' first, not enrolled in a conflicting class) and each classes' capacity. If i is quasi-sophisticated, then she expects s and s' to have the same number of students rank it first.

Note that a student who has symmetric information about others' reports and believes that their desired class sections are overdemanded will follow the strategy implied by Theorem 1. While the weighted selection implies that $\Pr(\mu(i) = s) \neq E\left[\min\left(\frac{q_s}{X_s+1}, 1\right)\right]$, as used in the proof of Theorem 1, a near-identical proof with a minor change to the "overdemanded" assumption (see Appendix D) holds if students are selected according to a weighting function. As a result, data from the ESP programs can be used to test the empirical benefit that a quasi-sophisticated strategy may have.

3.3 Preference Structure

In Table 4, we describe for each program the structure of observed student preferences and average matchings across 100 iterations of the ESP mechanism. On average, students tend to rank a class in most timeblocks. However, due to the Boston-like nature of the ESP mechanism and the low supply-demand ratio across all programs, students do not tend to be enrolled in many starred classes, particularly in Spring and Summer HSSP.

In this environment, it is plausible that students have symmetric information about all class sections. While there is significant heterogeneity in student preferences, as shown in Figure 1a, students have no basis on which to guess which classes are more desired. ESP organizers never communicate any information about which classes have more or fewer registrations to participants, before or after the lottery is run. Further, students also cannot easily learn about which classes are popular in one program and apply that knowledge to future programs, as the class catalog changes between every instance of ESP programs. As a result, it is reasonable to assume that any given student has symmetric information.

Further, it is reasonable for a student to use the quasi-sophisticated strategy described in Subsection 2.2. As the complexity of the ESP mechanism is quite large (the largest programs have nearly one thousand class sections and hundreds of thousands of stars), students are likely unprepared to perform the detailed calculations required to determine the optimal strategy, and may instead use the quasi-sophisticated strategy. However, the quasi-sophisticated strategy is likely *not* a good approximation of the fully-rational strategy in the ESP lottery, as shown in Figure 1b. There is significant heterogeneity in the popularity of sections, and only 54.4% of class sections are overdemanded, violating the assumption made by a quasi-sophisticated student. While students may follow the quasi-sophisticated strategy, it will not approximate a fully rational strategy well due to the large number of class sections that are underdemanded.

As it is plausible for a student to both follow the quasi-sophisticated strategy and to hold symmetric information, data from ESP provide an ideal environment to test the utility benefits of the strategy implied by Theorem 1.

4 Analysis of Strategic Behavior

4.1 Methodology

We analyze the relative benefits of truth-telling, the quasi-sophisticated strategy, and a fully-rational strategy with perfect information about others' preferences using the ESP data set. For each program, we run 600 trials, each with the data from all existing students and an additional, synthetic student. This student will have preferences only over two class sections s_1, s_2 with $s_1 \succ s_2$, with the two class sections chosen uniformly from all pairs of classes that happen in the same timeblock, have the same duration, and have some overlap in their permitted grade ranges. We endow our simulated student with $U(s_1) = 1$, $U(s_2) = 0.75$, and $U(i) = 0$. Our ratio of $U(s_2)/U(s_1) = 0.75$ is in line with the ratio used in experimental papers about school choice mechanisms, such as Chen and Sönmez (2006) ($U(s_1)/U(s_2) = 0.81$), Featherstone and Niederle (2009) ($U(s_1)/U(s_2) = 0.70$), and Pais et al. (2011) ($U(s_1)/U(s_2) = 0.60$).

For this synthetic student, we calculate their expected utility of truth-telling $U_{truth} = EU\left(\widehat{R_i} = s_1 \succ s_2\right)$ using 100 simulations of the ESP mechanism with true reported preferences, and their expected utility of reversing their preferences $U_{reverse} = EU\left(\widehat{R_i} = s_2 \succ s_1\right)$ using 100 simulations with reversed preferences. We calculate the students' expected utility of following the quasi-sophisticated strategy as $U_{QS} = U_{truth}$ if $U(s_1)q_1 \geq U(s_2)q_2$, else $U_{reverse}$. Finally, we calculate students' expected utility with perfect information as $U_{PI} = \max(U_{truth}, U_{reverse})$, as a rational agent with complete information will always choose the strategy which gives them higher utility in expectation.

4.2 Results

Our primary results can be found in Table 5a and Table 5b. However, the quasi-sophisticated strategy performs *worse* than the truthful strategy by 0.036 units, a difference which is statistically significant at the 5% level.[12] As such, the quasi-sophisticated strategy, while utility-improving in expectation, is detrimental to a students' outcome in the ESP mechanism.

We report the same results, grouped by program, in Table 6 and Table 7. In only 2 of the 28 programs — Spring HSSP 2016 and Summer HSSP 2018 — does the quasi-sophisticated strategy outperform truth-telling. However, neither of these differences are significant at the 5% level, implying that there is no statistical benefit if our simulated student follows the quasi-sophisticated strategy.

Further, an agent with full information outperforms an agent who is quasi-sophisticated (by 0.034 units) or a truth-telling agent (by 0.070 units). This difference is also statistically significant across all programs. This result highlights the extent to which the ESP mechanism is manipulable in this setting; however, manipulating does require detailed information about others' preferences. If an agent attempts to act strategically, but only possesses symmetric information, they cannot achieve the same level of utility as an agent who has access to others' reports.

[12] A magnitude of 0.036 units can be interpreted as an equivalent increase in the likelihood to be enrolled in a students' true first choice by 3.6 percentage points. Similar interpretations hold throughout this section.

To explain the inferior performance of the quasi-sophisticated strategy when compared to truth-telling, we refer to Figure 2, which contains histograms of enrollment into s_1 and s_2 under true and reversed preferences. For 49.3% of simulated students, reporting true preferences enrolls them into their s_1 100% of the time; under reversed preferences, 49.0% of simulated students are always enrolled into s_2. As was expected from information presented in Subsection 3.3, many class sections are *under* capacity. If your true first choice is under-capacity in the first round of the ESP mechanism, it is always optimal to report truthfully, as you are guaranteed enrollment in your first choice. In other words, the "overdemanded" assumption made by a student following the quasi-sophisticated strategy does not match the reality of the ESP environment. As a result, the quasi-sophisticated strategy performs poorly because it underestimates the likelihood of enrollment into a students' first choice, and so underestimates the benefits of truthfulness.

Could the quasi-sophisticated strategy have more merit in a more capacity-constrained economy? To simulate such an environment, we repeat our analysis excluding the 12,208 students from our simulations who were either always enrolled in s_1 under their true preferences or always enrolled in s_2 under reversed preferences; in essence, restricting our sample to students who only have preferences for overdemanded class sections. This process leaves 4,592 simulated students; for them, the quasi-sophisticated strategy approximates in theory the fully-rational strategy (see Section 2). We report our results for this subgroup in Table 8a and Table 8b, and results grouped by program in Table 9 and Table 10.

Even in this sample limited to situations that suit the quasi-sophisticated strategy, it still does no better than truth-telling. In this subgroup, the difference between U_{true} and U_{QS} is 0.002 units; while the quasi-sophisticated strategy still performs worse than the truth-telling strategy, the difference is no longer statistically significant. Twelve of the 28 programs have an expected utility by following the quasi-sophisticated strategy U_{QS} than by truth-telling

U_{Truth}, and in only two of these programs does the quasi-sophisticated strategy outperform the truth-telling strategy to an extent that is statistically significant at the 5% level. In all cases, a student with full information outperforms a student using the quasi-sophisticated strategy at the 5% level.

To explain the limited usefulness of the quasi-sophisticated strategy, we note that there is still a significant amount of heterogeneity in the likelihood of enrollment into a class section when ranking it first, as shown in Figure 3. Under symmetric information, a quasi-sophisticate expects to be enrolled into each class with the same probability, as demonstrated in the proof of Theorem 1. However, it may be the case that the *realization* of preferences does not result in equal popularity for all classes. In such cases, the quasi-sophisticated strategy misjudges the likelihood of enrollment into classes, and so empirically performs no better than truth-telling.

5 Conclusion

If a student in an overdemanded environment holds symmetric information, there are theoretical benefits to manipulation by using the quasi-sophisticated strategy. In practice, however, these manipulations do not improve a student's expected outcome compared to truth-telling in the environment of the Educational Studies Program. This null result is not just driven by sections that are not overdemanded, but is also driven by the wide heterogeneity in the realization of other students' preferences. In particular, a *lack* of information may lead students to be honest, even under highly manipulable mechanisms. We also find that both truth-telling and the quasi-sophisticated strategy are dominated by the strategy taken by an agent who has full knowledge about student preferences, a result that highlights the importance of information in manipulating the Boston mechanism. These results suggest that manipulable mechanisms become *less* manipulable if information about student and school preferences are withheld.

There are many potential avenues for future work in this area. While this paper has examined the individual incentives to truth-tell using the quasi-sophisticated strategy, we do not examine the theoretical or empirical effects that quasi-sophisticated students may have across *all* students. Further, we do not examine how a lack of information affects the distribution of assignments across students, especially as it relates to the equity of matches for sophisticated and non-sophisticated students. While the work of economists has caused many school districts to replace the Boston mechanism with strategy-proof alternatives, given that the Boston mechanism is still present in many markets today, including the ESP environment, it is important that economists understand under what circumstances individuals face incentives to strategically manipulate, and when those manipulations succeed.

Appendices

A Tables

Table 1: Expected utility in Example 1, conditional on others' reports

Preferences of students i_2, i_3, i_4	$E\left[U \mid \widehat{R_{i_1}} = (s_1 \succ s_2)\right]$	$E\left[U \mid \widehat{R_{i_1}} = (s_1 \succ s_2)\right]$
$3 \times (s_1 \succ s_2),\ 0 \times (s_2 \succ s_1)$	$\frac{1}{4} \cdot 1 + \frac{3}{4}\left(\frac{2}{3}U(s_2)\right)$	$U(s_2)$
$2 \times (s_1 \succ s_2),\ 1 \times (s_2 \succ s_1)$	$\frac{1}{3} \cdot 1 + \frac{2}{3}\left(\frac{1}{2}U(s_2)\right)$	$U(s_2)$
$1 \times (s_1 \succ s_2),\ 2 \times (s_2 \succ s_1)$	$\frac{1}{2} \cdot 1 + \frac{1}{2}(0)$	$\frac{2}{3}U(s_2) + \frac{1}{3}(0)$
$0 \times (s_1 \succ s_2),\ 3 \times (s_2 \succ s_1)$	1	$\frac{1}{2}U(s_2) + \frac{1}{2}\left(\frac{1}{2} \cdot 1\right)$

Table 2: Expected utility in Example 2, conditional on others' reports

Preferences of students i_2, i_3, i_4	$E\left[U \mid \widehat{R_{i_1}} = (s_1 \succ s_2)\right]$	$E\left[U \mid \widehat{R_{i_1}} = (s_1 \succ s_2)\right]$
$3 \times (s_1 \succ s_2),\ 0 \times (s_2 \succ s_1)$	$\frac{1}{4} \cdot 1$	$U(s_2)$
$2 \times (s_1 \succ s_2),\ 1 \times (s_2 \succ s_1)$	$\frac{1}{3} \cdot 1$	$U(s_2)$
$1 \times (s_1 \succ s_2),\ 2 \times (s_2 \succ s_1)$	$\frac{1}{2} \cdot 1$	$\frac{2}{3}U(s_2)$
$0 \times (s_1 \succ s_2),\ 3 \times (s_2 \succ s_1)$	1	$\frac{1}{2}U(s_2)$

Table 3: Summary statistics about ESP programs

	Counts				Capacities				
Program	Students	Sections	Classes	Timeblocks	Mean	SD	Median	Student-Hours	S/D Ratio
Spring HSSP 2014	455	45	43	4	28.4	27.4	24	1415	0.78
Spring HSSP 2015	566	48	48	4	28.9	27.3	25	1725	0.76
Spring HSSP 2016	588	38	37	4	28.5	30.6	21.5	1229	0.52
Spring HSSP 2017	613	39	34	6	28.7	29.8	20	2866	0.78
Spring HSSP 2018	643	33	32	6	31.0	25.2	24	2530	0.66
Spring HSSP 2019	536	36	34	6	32.9	25.2	30	2674	0.83
Spring HSSP 2020	827	45	44	6	36.6	26.7	30	3986	0.80
Summer HSSP 2014	760	41	36	4	49.8	44.3	30	2277	0.75
Summer HSSP 2015	1124	61	60	5	34.3	31.9	25	2567	0.46
Summer HSSP 2016	1021	54	45	4	28.8	19.0	24	1556	0.38
Summer HSSP 2017	983	49	44	3	29.7	20.7	25	1454	0.49
Summer HSSP 2018	1138	53	51	4	35.2	30.2	27	1863	0.41
Summer HSSP 2019	914	53	50	10	29.6	19.7	25	3981	0.44
Summer HSSP 2020	2500	128	119	10	61.2	63.1	30	17301	0.69
Splash 2014	3261	842	568	19	40.9	56.3	25	51443	0.83
Splash 2015	3012	898	588	19	43.4	71.7	25	55184	0.96
Splash 2016	2858	868	559	19	46.9	67.6	30	56477	1.04
Splash 2017	2483	681	433	19	42.5	65.4	27	41741	0.88
Splash 2018	2201	550	360	19	49.0	74.5	30	39251	0.94
Splash 2019	2187	726	474	19	41.9	69.6	25	40430	0.97
Splash 2020	1318	293	200	19	49.8	51.0	30	19825	0.79
Spark 2014	1209	311	213	13	39.9	59.3	25	15961	1.02
Spark 2015	1308	310	209	14	45.4	70.0	27	18084	0.99
Spark 2016	1566	328	204	14	38.5	44.0	29.5	15808	0.72
Spark 2017	1491	291	186	15	40.9	40.7	25	14137	0.63
Spark 2018	1324	210	138	15	60.8	93.7	30	16344	0.82
Spark 2019	1317	271	186	15	50.6	92.7	25	17921	0.91
Spark 2020	1264	256	176	15	36.5	42.9	25	12542	0.66

Notes: Table includes information about all student reports registered in the ESP Website. Statistics about class sections and capacities include only sections for which at least one student marked interest (rank or star), and excludes sections with the category “Lunch”, a category used by ESP to assign students to lunch blocks.
Source: Educational Studies Program data set.

Table 4: Summary statistics about student preferences and enrollment

	Average Student Preferences			Average Student Enrollment			
Program	1^{st}	Stars	Stars excluding 1^{st}	1^{st}	Stars	Stars excluding 1^{st}	Total
Spring HSSP 2014	2.70	5.72	3.56	1.69	1.78	0.43	2.13
Spring HSSP 2015	2.45	6.18	4.15	1.43	1.67	0.46	1.89
Spring HSSP 2016	2.42	5.29	3.38	1.18	1.26	0.33	1.51
Spring HSSP 2017	2.75	7.22	4.83	1.14	1.28	0.28	1.42
Spring HSSP 2018	2.35	6.24	4.22	0.95	1.18	0.35	1.30
Spring HSSP 2019	2.13	8.66	6.73	1.10	1.71	0.72	1.82
Spring HSSP 2020	2.75	9.05	6.65	1.27	1.51	0.39	1.67
Summer HSSP 2014	3.02	7.06	4.63	1.66	1.87	0.52	2.18
Summer HSSP 2015	2.44	7.49	5.55	1.20	1.36	0.41	1.61
Summer HSSP 2016	3.10	8.55	5.98	1.22	1.27	0.28	1.50
Summer HSSP 2017	2.48	7.96	5.88	1.09	1.28	0.35	1.45
Summer HSSP 2018	3.22	10.56	7.80	1.22	1.43	0.39	1.61
Summer HSSP 2019	3.50	12.07	8.96	1.24	1.32	0.21	1.45
Summer HSSP 2020	3.60	16.57	13.26	1.73	2.38	0.79	2.52
Splash 2014	13.00	76.86	65.38	5.16	6.84	2.30	7.46
Splash 2015	13.25	67.10	55.83	5.74	7.13	2.28	8.02
Splash 2016	12.23	70.94	60.24	5.60	7.40	2.53	8.14
Splash 2017	11.72	55.18	45.26	5.23	6.55	2.10	7.34
Splash 2018	11.71	61.57	51.55	5.46	7.17	2.49	7.95
Splash 2019	13.61	84.57	72.51	6.51	8.67	2.89	9.41
Splash 2020	10.37	40.16	31.25	5.21	6.48	2.02	7.23
Spark 2014	9.65	36.28	28.45	4.55	5.30	1.62	6.18
Spark 2015	10.19	38.31	29.86	4.68	5.45	1.58	6.26
Spark 2016	10.17	41.57	33.26	4.27	5.10	1.68	5.95
Spark 2017	11.42	49.58	39.99	4.82	5.91	1.91	6.73
Spark 2018	11.36	47.69	38.04	4.41	5.58	1.86	6.27
Spark 2019	11.77	53.17	43.12	4.70	5.74	1.75	6.46
Spark 2020	12.74	56.12	45.38	4.76	5.26	1.25	6.02

Notes: Columns under "Average Student Preferences" indicate the average number of classes that each student ranked first, starred, and starred but excluding first choices. We distinguish between starred classes and starred excluding first choices, as students can both star a class and rank it first. Columns under "Average Student Enrollment" indicate the average number of classes that each student was enrolled into which were ranked first by the student, starred by the student, or starred by the student excluding they also ranked first, calculated using 100 simulations of the ESP Lottery.
Source: Educational Studies Program dataset, simulations performed by author.

Table 5: Simulation results of various strategies in the ESP mechanism

(a) Average utilities.

	Mean	Standard Error
U_{truth}	0.835	0.002
U_{QS}	0.799	0.002
U_{FI}	0.869	0.001
N	16800	

(b) p-values of comparisons.

	p-value
H_a: $U_{QS} > U_{truth}$	1.000
H_a: $U_{FI} > U_{truth}$	0.000
H_a: $U_{FI} > U_{QS}$	0.000

Notes: Utilities calculated using 600 simulated students for each of 28 programs. Each simulated student has preferences over a pair of class sections, the pair chosen uniformly over all pairs of class sections that occur in the same timeblock and have the same length. p-values are one-sided paired $t-$tests for the alternative hypotheses listed in the table.
Source: Educational Studies Program data set, simulations performed by author.

Table 6: Means and standard errors of various strategies in the ESP mechanism, grouped by program

	U_{truth}		U_{QS}		U_{FI}		
Program	Mean	SE	Mean	SE	Mean	SE	N
Spring HSSP 2014	0.888	0.008	0.834	0.008	0.900	0.007	600
Spring HSSP 2015	0.889	0.008	0.857	0.008	0.910	0.006	600
Spring HSSP 2016	0.757	0.012	0.761	0.010	0.819	0.008	600
Spring HSSP 2017	0.805	0.011	0.788	0.010	0.857	0.008	600
Spring HSSP 2018	0.852	0.011	0.804	0.010	0.886	0.008	600
Spring HSSP 2019	0.944	0.006	0.907	0.006	0.954	0.004	600
Spring HSSP 2020	0.844	0.009	0.807	0.009	0.862	0.008	600
Summer HSSP 2014	0.781	0.011	0.752	0.010	0.817	0.009	600
Summer HSSP 2015	0.779	0.011	0.724	0.010	0.816	0.008	600
Summer HSSP 2016	0.645	0.013	0.641	0.012	0.722	0.011	600
Summer HSSP 2017	0.721	0.012	0.701	0.011	0.785	0.009	600
Summer HSSP 2018	0.656	0.012	0.662	0.011	0.718	0.010	600
Summer HSSP 2019	0.644	0.012	0.633	0.012	0.724	0.010	600
Summer HSSP 2020	0.842	0.010	0.805	0.009	0.874	0.007	600
Spark 2014	0.921	0.007	0.871	0.007	0.934	0.006	600
Spark 2015	0.893	0.008	0.844	0.007	0.914	0.006	600
Spark 2016	0.885	0.008	0.832	0.008	0.905	0.006	600
Spark 2017	0.828	0.010	0.793	0.009	0.862	0.008	600
Spark 2018	0.799	0.011	0.779	0.009	0.843	0.008	600
Spark 2019	0.856	0.009	0.817	0.008	0.892	0.007	600
Spark 2020	0.809	0.010	0.781	0.009	0.848	0.008	600
Splash 2014	0.891	0.008	0.845	0.007	0.914	0.006	600
Splash 2015	0.899	0.008	0.843	0.007	0.926	0.005	600
Splash 2016	0.938	0.006	0.872	0.006	0.950	0.004	600
Splash 2017	0.910	0.008	0.862	0.007	0.932	0.005	600
Splash 2018	0.898	0.008	0.844	0.007	0.922	0.006	600
Splash 2019	0.912	0.007	0.855	0.007	0.931	0.005	600
Splash 2020	0.901	0.007	0.848	0.007	0.916	0.005	600

Notes: Utilities calculated using 600 simulated students for each of 28 programs. Each simulated student has preferences over a pair of class sections, the pair chosen uniformly over all pairs of class sections that occur in the same timeblock and have the same length.
Source: Educational Studies Program data set, simulations performed by author.

Table 7: Comparisons of various strategies in the ESP mechanism, grouped by program

	Utility Differences			p-values		
Program	$U_{QS} - U_{truth}$	$U_{FI} - U_{truth}$	$U_{FI} - U_{QS}$	$U_{QS} > U_{truth}$	$U_{FI} > U_{truth}$	$U_{FI} > U_{QS}$
Spring HSSP 2014	-0.055	0.012	0.067	1	0	0
Spring HSSP 2015	-0.032	0.022	0.054	1	0	0
Spring HSSP 2016	0.004	0.062	0.058	0.270	0	0
Spring HSSP 2017	-0.009	0.054	0.063	0.980	0	0
Spring HSSP 2018	-0.047	0.034	0.081	1	0	0
Spring HSSP 2019	-0.038	0.009	0.047	1	0	0
Spring HSSP 2020	-0.037	0.018	0.055	1	0	0
Summer HSSP 2014	-0.029	0.036	0.064	1	0	0
Summer HSSP 2015	-0.055	0.037	0.091	1	0	0
Summer HSSP 2016	-0.003	0.077	0.080	0.692	0	0
Summer HSSP 2017	-0.020	0.064	0.083	0.998	0	0
Summer HSSP 2018	0.006	0.062	0.055	0.172	0	0
Summer HSSP 2019	-0.010	0.080	0.090	0.950	0	0
Summer HSSP 2020	-0.037	0.032	0.069	1	0	0
Spark 2014	-0.050	0.013	0.062	1	0	0
Spark 2015	-0.049	0.021	0.070	1	0	0
Spark 2016	-0.052	0.020	0.072	1	0	0
Spark 2017	-0.036	0.033	0.069	1	0	0
Spark 2018	-0.020	0.044	0.064	0.998	0	0
Spark 2019	-0.039	0.037	0.076	1	0	0
Spark 2020	-0.028	0.039	0.067	1	0	0
Splash 2014	-0.047	0.023	0.070	1	0	0
Splash 2015	-0.056	0.027	0.083	1	0	0
Splash 2016	-0.065	0.012	0.078	1	0	0
Splash 2017	-0.048	0.022	0.070	1	0	0
Splash 2018	-0.054	0.024	0.078	1	0	0
Splash 2019	-0.057	0.019	0.076	1	0	0
Splash 2020	-0.053	0.014	0.068	1	0	0

Notes: Utilities calculated using 600 simulated students for each of 28 programs. Each simulated student has preferences over a pair of class sections, the pair chosen uniformly over all pairs of class sections that occur in the same timeblock and have the same length. p-values are one-sided paired $t-$tests for the alternative hypotheses listed in the table.
Source: Educational Studies Program data set, simulations performed by author.

Table 8: Simulation results of various strategies in the ESP mechanism for oversubscribed sections

(a) Average utilities.

	Mean	Standard Error
U_{truth}	0.578	0.004
U_{QS}	0.576	0.004
U_{FI}	0.646	0.003
N	4592	

(b) p-values of comparisons.

	p-value
H_a: $U_{QS} > U_{truth}$	0.802
H_a: $U_{FI} > U_{truth}$	0.000
H_a: $U_{FI} > U_{QS}$	0.000

Notes: Utilities calculated using 600 simulated students for each of 28 programs, excluding 12,208 simulated students that are always enrolled in their reported first choice under either true or reversed preferences. Each simulated student has preferences over a pair of class sections, the pair chosen uniformly over all pairs of class sections that occur in the same timeblock and have the same length. p-values are one-sided paired $t-$tests for the alternative hypotheses listed in the table.
Source: Educational Studies Program data set, simulations performed by author.

Table 9: Means and standard errors of various strategies in the ESP mechanism for oversubscribed sections, grouped by program

	U_{truth}		U_{QS}		U_{FI}		
Program	Mean	SE	Mean	SE	Mean	SE	N
Spring HSSP 2014	0.611	0.020	0.574	0.021	0.651	0.018	118
Spring HSSP 2015	0.529	0.026	0.584	0.024	0.622	0.020	87
Spring HSSP 2016	0.520	0.018	0.562	0.015	0.633	0.012	237
Spring HSSP 2017	0.490	0.026	0.504	0.020	0.595	0.021	140
Spring HSSP 2018	0.508	0.029	0.477	0.029	0.601	0.024	121
Spring HSSP 2019	0.564	0.035	0.532	0.044	0.612	0.033	30
Spring HSSP 2020	0.537	0.018	0.549	0.017	0.593	0.015	149
Summer HSSP 2014	0.548	0.017	0.526	0.015	0.602	0.014	214
Summer HSSP 2015	0.556	0.015	0.554	0.013	0.617	0.011	235
Summer HSSP 2016	0.442	0.014	0.447	0.014	0.527	0.012	300
Summer HSSP 2017	0.465	0.015	0.443	0.015	0.522	0.011	193
Summer HSSP 2018	0.485	0.014	0.491	0.012	0.555	0.012	331
Summer HSSP 2019	0.504	0.014	0.488	0.013	0.588	0.012	341
Summer HSSP 2020	0.549	0.021	0.542	0.018	0.628	0.016	148
Spark 2014	0.688	0.028	0.703	0.024	0.731	0.023	100
Spark 2015	0.654	0.023	0.659	0.019	0.708	0.018	117
Spark 2016	0.670	0.021	0.637	0.021	0.708	0.018	131
Spark 2017	0.607	0.018	0.613	0.016	0.671	0.014	197
Spark 2018	0.555	0.020	0.573	0.018	0.626	0.017	182
Spark 2019	0.672	0.020	0.677	0.017	0.743	0.015	173
Spark 2020	0.651	0.017	0.641	0.016	0.713	0.014	250
Splash 2014	0.682	0.024	0.679	0.021	0.740	0.017	122
Splash 2015	0.720	0.024	0.702	0.021	0.781	0.017	118
Splash 2016	0.698	0.030	0.690	0.026	0.764	0.020	69
Splash 2017	0.691	0.026	0.695	0.022	0.754	0.019	103
Splash 2018	0.694	0.023	0.693	0.021	0.748	0.019	116
Splash 2019	0.730	0.023	0.719	0.018	0.780	0.017	119
Splash 2020	0.756	0.016	0.733	0.015	0.785	0.013	151

Notes: Utilities calculated using 600 simulated students for each of 28 programs, excluding 12,208 simulated students that are always enrolled in their reported first choice under either true or reversed preferences. Each simulated student has preferences over a pair of class sections, the pair chosen uniformly over all pairs of class sections that occur in the same timeblock and have the same length.
Source: Educational Studies Program data set, simulations performed by author.

Table 10: Comparisons of various strategies in the ESP mechanism for oversubscribed sections, grouped by program

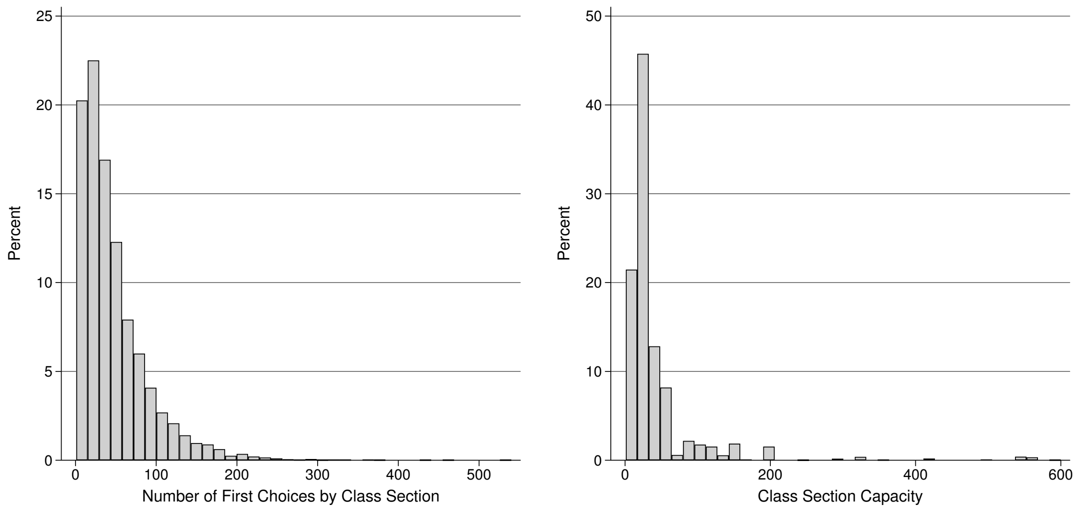
	Utility Differences			p-values		
Program	$U_{QS} - U_{truth}$	$U_{FI} - U_{truth}$	$U_{FI} - U_{QS}$	$U_{QS} > U_{truth}$	$U_{FI} > U_{truth}$	$U_{FI} > U_{QS}$
Spring HSSP 2014	-0.036	0.040	0.077	0.993	0	0
Spring HSSP 2015	0.056	0.093	0.037	0.001	0	0
Spring HSSP 2016	0.042	0.113	0.071	0	0	0
Spring HSSP 2017	0.014	0.105	0.091	0.276	0	0
Spring HSSP 2018	-0.031	0.093	0.124	0.949	0	0
Spring HSSP 2019	-0.032	0.048	0.080	0.892	0.004	0
Spring HSSP 2020	0.012	0.056	0.044	0.122	0	0
Summer HSSP 2014	-0.022	0.055	0.076	0.980	0	0
Summer HSSP 2015	-0.002	0.061	0.063	0.573	0	0
Summer HSSP 2016	0.005	0.084	0.079	0.274	0	0
Summer HSSP 2017	-0.021	0.057	0.079	0.979	0	0
Summer HSSP 2018	0.007	0.070	0.064	0.239	0	0
Summer HSSP 2019	-0.017	0.084	0.100	0.980	0	0
Summer HSSP 2020	-0.007	0.079	0.086	0.692	0	0
Spark 2014	0.014	0.043	0.028	0.127	0	0
Spark 2015	0.005	0.054	0.049	0.338	0	0
Spark 2016	-0.033	0.038	0.071	0.991	0	0
Spark 2017	0.006	0.065	0.058	0.282	0	0
Spark 2018	0.019	0.071	0.052	0.054	0	0
Spark 2019	0.005	0.071	0.066	0.366	0	0
Spark 2020	-0.010	0.062	0.072	0.831	0	0
Splash 2014	-0.003	0.058	0.061	0.595	0	0
Splash 2015	-0.018	0.061	0.079	0.886	0	0
Splash 2016	-0.008	0.066	0.074	0.618	0	0
Splash 2017	0.004	0.063	0.059	0.413	0	0
Splash 2018	-0.001	0.054	0.055	0.530	0	0
Splash 2019	-0.011	0.051	0.061	0.725	0	0
Splash 2020	-0.023	0.029	0.052	0.986	0	0

Notes: Utilities calculated using 600 simulated students for each of 28 programs, excluding 12,208 simulated students that are always enrolled in their reported first choice under either true or reversed preferences. Each simulated student has preferences over a pair of class sections, the pair chosen uniformly over all pairs of class sections that occur in the same timeblock and have the same length. p-values are one-sided paired $t-$tests for the alternative hypotheses listed in the table.
Source: Educational Studies Program data set, simulations performed by author.

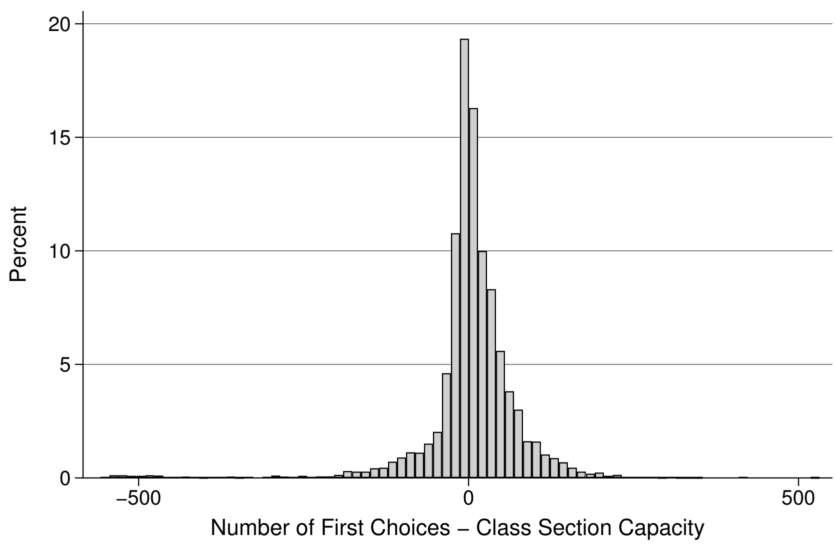
B Figures

Figure 1: Histograms of class sections

(a) Histograms of first choices and class section capacities.



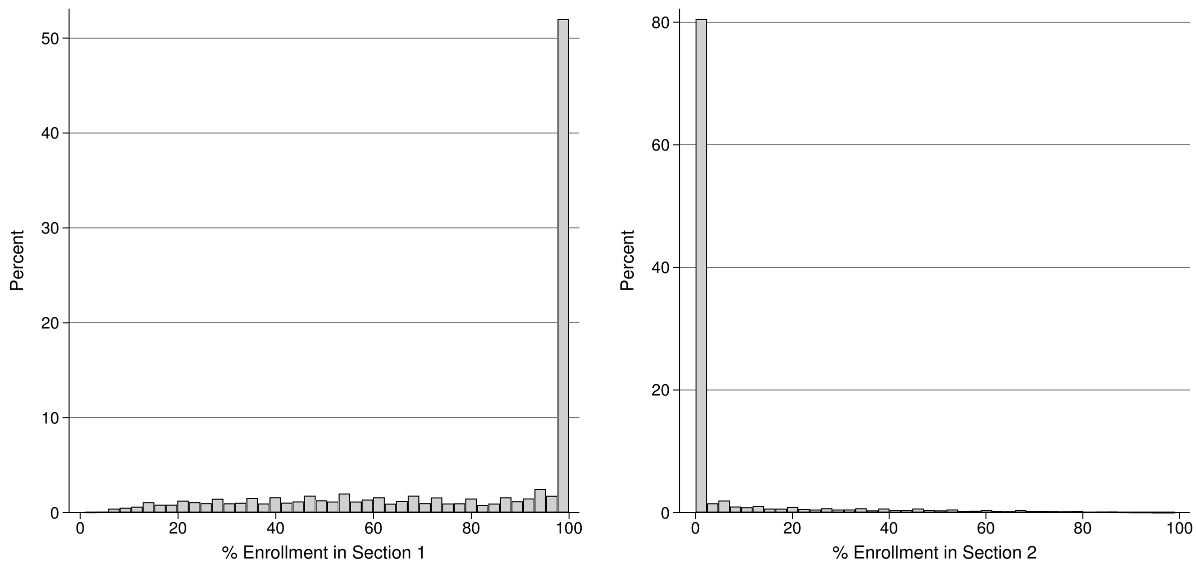
(b) Histogram of the difference between number of first choices and capacity.



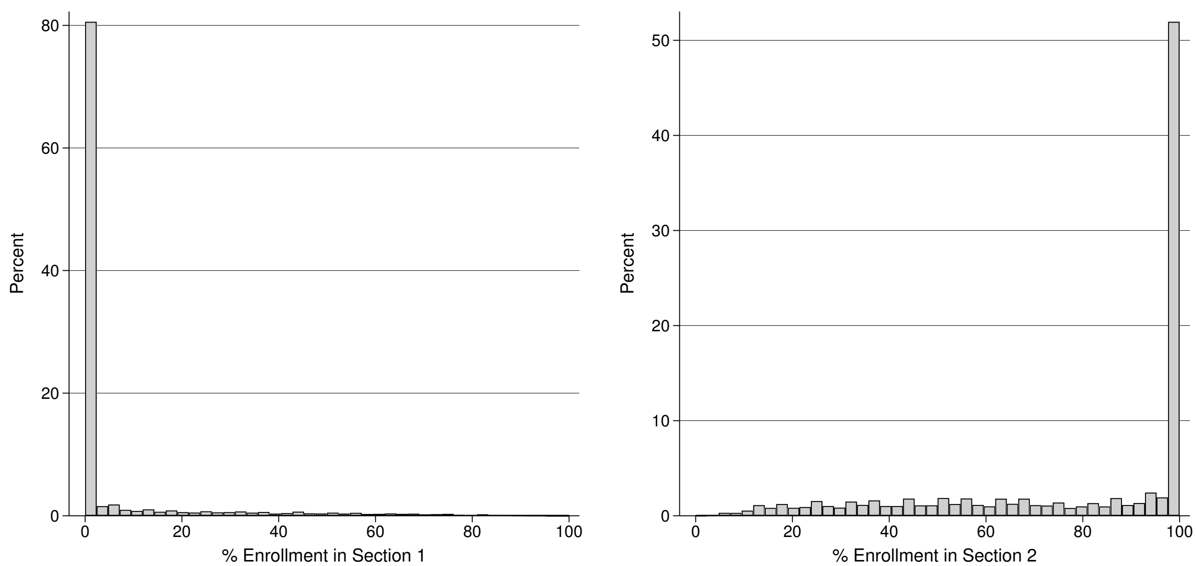
Figures: Table includes information about all student reports registered in the ESP Website. Statistics about class sections and capacities include only sections for which at least one student marked interest (rank or star), and excludes sections with the category "Lunch", a category used by ESP to assign students to lunch blocks.
Source: Educational Studies Program data set.

Figure 2: Enrollment into class sections under various reports

(a) Enrollment under true preferences



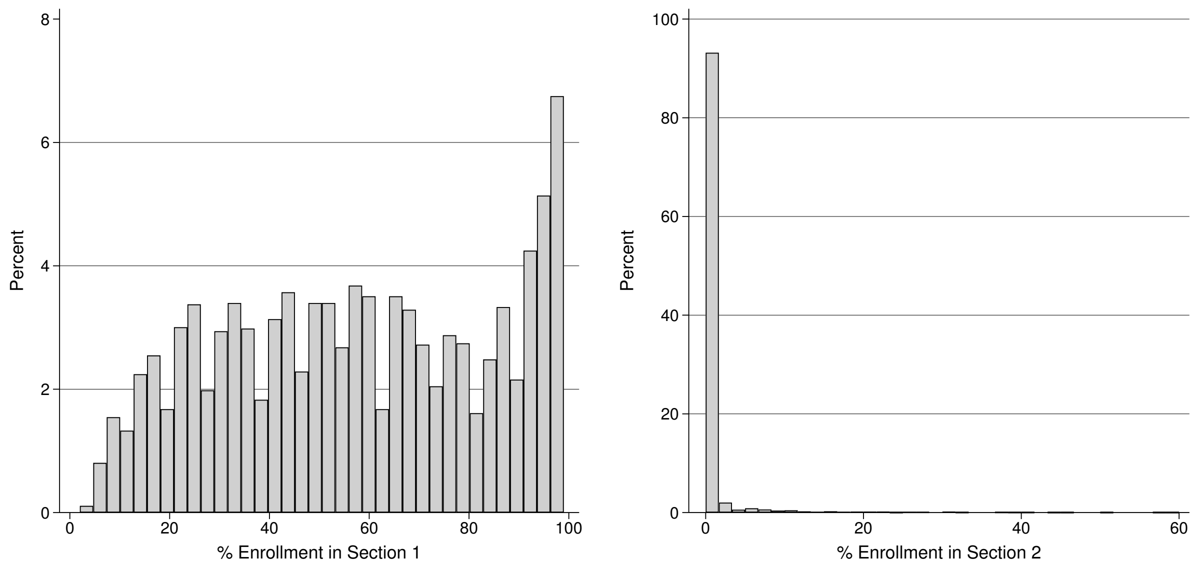
(b) Enrollment under reversed preferences



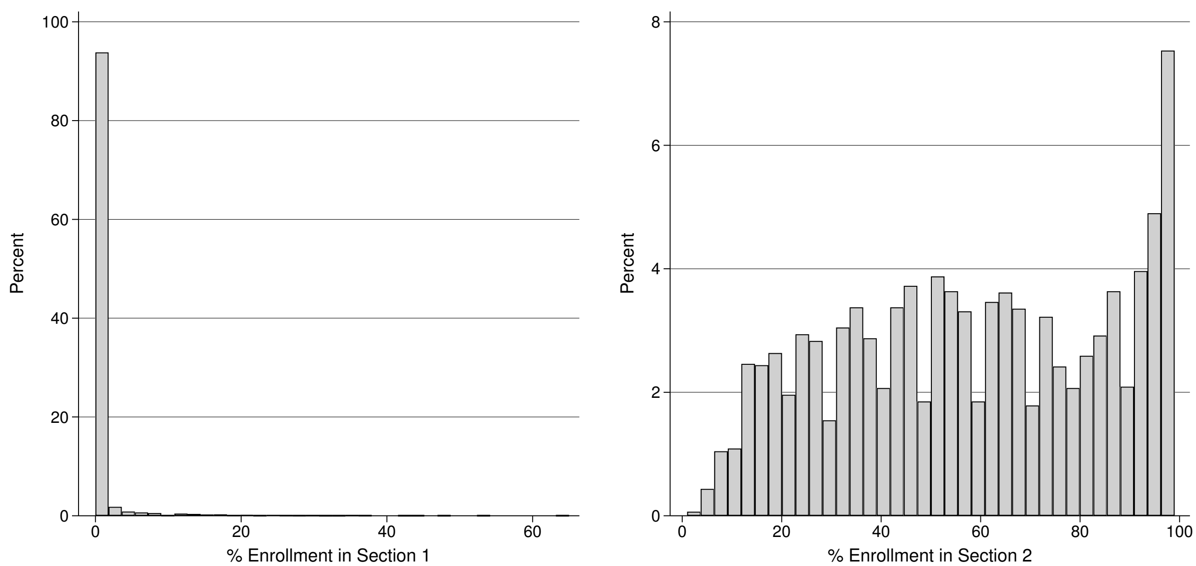
Notes: Utilities calculated using 600 simulated students for each of 28 programs. Each simulated student has preferences over a pair of class sections, the pair chosen uniformly over all pairs of class sections that occur in the same timeblock and have the same length.
Source: Educational Studies Program data set, simulations performed by author.

Figure 3: Enrollment into class sections under various reports for oversubscribed sections

(a) Enrollment under true preferences



(b) Enrollment under reversed preferences



Notes: Utilities calculated using 600 simulated students for each of 28 programs, excluding 12,208 simulated students that are always enrolled in their reported first choice under either true or reversed preferences. Each simulated student has preferences over a pair of class sections, the pair chosen uniformly over all pairs of class sections that occur in the same timeblock and have the same length.
Source: Educational Studies Program data set, simulations performed by author.

C Example of Quasi-Sophisticated Behavior in Larger Markets

Example 3. Consider a school choice market with the same setup as Examples 1 and 2, where a student i has preferences over schools s_1, s_2, with capacities 1 and 2, respectively, such that $U(s_1) = 1 > U(s_2) > U(i) = 0$. However, let there be 10 students other than i with preferences drawn uniformly from $\{(s_1 \succ s_2), (s_2 \succ s_1)\}$. If i is fully rational, he believes his probability of enrollment in s_1 and s_2 under various reports , conditional on others' preferences, to be as follows:

Table 11: Probabilities of enrollment in s_1, s_2 under truth-telling and reversal strategies in Example 3.

Others' Preferences		Truth: $\widehat{R}_i = (s_1 \succ s_2)$		Reversal: $\widehat{R}_i = (s_2 \succ s_1)$	
# of $s_1 \succ s_2$	# of $s_2 \succ s_1$	$\Pr(\mu(i) = s_1)$	$\Pr(\mu(i) = s_2)$	$\Pr(\mu(i) = s_1)$	$\Pr(\mu(i) = s_2)$
10	0	0.091	0.182	0	1
9	1	0.100	0.100	0	1
8	2	0.111	0	0	0.667
7	3	0.125	0	0	0.500
6	4	0.143	0	0	0.400
5	5	0.167	0	0	0.333
4	6	0.200	0	0	0.286
3	7	0.250	0	0	0.250
2	8	0.333	0	0	0.222
1	9	0.500	0	0	0.200
0	10	1	0	0.091	0.182

Note that row n (starting with $n = 0$) occurs with probability $0.5^{10}\binom{10}{n}$. As a result, a fully-rational agent calculates their expected utility of truth-telling as $0.1817 + 0.0012U(s_2)$ and expected utility of reporting her preference in reverse order as $0.00008 + 0.3625U(s_2)$.

She will then choose to report false preferences if and only if $U(s_2) > 0.5027$.

A quasi-sophisticated agent believes his probability of enrollment into his second choice (i.e, columns 4 and 5) to be all 0. He calculates his expected utility of truth-telling as 0.1817 and expected utility of reporting his preferences in reverse order as $0.3625U(s_2)$. He will choose to report false preferences if and only if $U(s_2) > 0.5013$.

In this example, the difference in cutoffs between a quasi-sophisticated and rational agent differ by 0.2%, or less than 1 part in 370. As such, the quasi-sophisticated strategy is a good approximation of a fully-rational agent.

Further, the cutoffs of both agents are very close to 0.5, the cutoff implied by Theorem 1 of $U(s_2) > \frac{U(s_1)q_{s_1}}{q_{s_2}} = \frac{1}{2}$. This is because the probability that s_1 or s_2 still has remaining capacity after 1 round is very small – either s_1 must receive no first choices or s_2 must receive fewer than 2 first choices, which occurs with probability $\frac{3}{256} \approx 0$. $\square$

D Modifying Theorem 1 for the ESP Mechanism

Suppose that student i ranks s first under the ESP mechanism. When the algorithm assigns students to s who selected s at their first choice, there will be some subset of students $J \subseteq I$ with weights $w = \{w_1, w_2, \cdots, w_i, \cdots, w_{|J|}\}$ who are being considered. So long as $\frac{w_j}{\sum w'} \leq \frac{1}{q_s} \ \forall \ j \in J$, a condition needed so that we do not run into edge cases, the probability that i is selected is given by $q_s \frac{w'_i}{\sum w'} \leq 1$. Another way to write this assumption is to renormalize w to w' such that $w'_i = 1$, and then let $Y_s = \sum w' - 1$ be a random variable over all possible realizations of the lottery which indicates the "effective" number of students that i is competing against for enrollment, excluding himself; then, our condition is equivalent to stating that $P(Y_s \geq q_s) = 1$. This condition is in the same spirit as the existing "overenrollment" assumption, but needed given the weighting and randomization procedure present in the ESP mechanism.

We prove the following:

Theorem 2. Consider a school choice market in which seats are assigned using the ESP Mechanism. Class sections s and s' occur during the same timeblock and have the same duration. Suppose that i has $\{s, s'\}$-symmetric information, i believes for each section s that Y_s, as defined above, satisfies $P(Y_s \geq q_s) = 1$, and that i believes that each school's priority ranking $\mathcal{R}_S$ is a random ordering of I.

If i has preferences $s \succ s'$, derives no utility from being unmatched, and follows the quasi-sophisticated strategy, she will choose to rank s_1 first if and only if $U(s)q_s \geq U(s')q_{s'}$; else, she benefits by reporting s' first.

Proof. The probability of enrollment in s when ranking it first is s is $\Pr(\mu(i) = s) = E\left[\min\left(\frac{q_s}{Y_s+1}, 1\right)\right]$. Symmetric information implies that the distributions of Y_s and $Y_{s'}$ coincide, and the proof proceeds the same way as the proof of Theorem 1. □

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